

## Small area estimation of the Quantile Ratio inequality Index

### Background

A significant increase in GDP per capita in a country may indicate economic national growth, but in reality, this growth might benefit only specific industrial sectors, regions or household groups, rather than the entire economy. It is well-established that growth is often accompanied by rising inequality, and that the inequality can diminish the positive impact of growth on absolute poverty reduction (Bourguignon, 2004). Inequality can also slow down growth itself, as lower-income households may become less productive (Aiyar and Ebeke, 2020). It is undesirable for both people and policy makers that growth benefits only a few, at the expense of increasing disparities in well-being. Reducing inequality, as emphasized by the 2030 Agenda for Sustainable Development, adopted by the United Nations (United Nations, 2015), is essential to ensure that economic growth effectively reduces poverty and improves living standards for all households.

In this socio-economic context, mapping inequality, particularly income inequality, which is of primary concern as highlighted by Piketty (2014), is crucial for reducing it. Disparities could be more pronounced in certain areas, for example cities where economic activities are concentrated, requiring special attention from governments. Many indicators exist in the literature offering different views of the income inequality. The choice of an indicator often depends on the researcher's focus and priorities. The Gini coefficient is certainly the most well-known and used measure of income concentration, due to its simplicity and the easy graphical representation with the Lorenz curve. A compelling alternative to the Gini coefficient, which has not yet been widely applied, is the Quantile Ratio Index (QRI). The QRI was originally proposed by Prendergast and Staudte (2018) as an inequality measure based on the ratio of symmetric quantiles, considering infinite (hyper-)populations, which also allows for an intuitive graphical representation of inequality.

This work aims at first at investigating small area estimation methods for the Quantile Ratio Index, for which, as far as we know, no literature exists. Secondly the performances of the SAE estimators of the Gini and of the QRI will be compared.

Small areas refer to domains where sample sizes are very low or even zero, making direct estimators potentially unreliable. Small area estimation (SAE) models can address this, by combining survey data with auxiliary data information, typically from census or administrative sources. These methods use explicit models to “borrow strength” from related areas and produce model-based estimators that have higher precision level. SAE methods have been widely studied in the literature (for reviews see Pfeiffermann (2013), Rao and Molina (2015), Morales et al. (2021)). Small area estimators of inequality indicators have received less attention than those for poverty. Tzavidis and Marchetti (2016) proposed an M-quantile regression model for the Quintile Share Ratio. Fabrizi and Trivisano (2016) proposed a hierarchical Bayesian Beta mixed model at the area-level for the estimation of the Gini coefficient. Marchetti and Tzavidis (2021) proposed for the same coefficient small area estimation models using M-quantile regression. De Nicolò et al. (2024a) adopted a Beta mixture-based approach, in a Bayesian framework, for estimating income inequality indicators in a unit-interval.

**Aim and deliveries**

This study aims to compare the model-based predictor of the Quantile Ratio Index (QRI) with that of the Gini coefficient, one of the most widely used income inequality indicators. The performance of the Small Area Estimation (SAE) estimators will be assessed through simulations, which will be conducted after generating a pseudo-population, following the methodology of Alfons et al. (2011). The analysis will be based on the 2020 Italian European Union - Statistics on Income and Living Conditions (EU-SILC) data, coordinated by Eurostat and conducted annually by the National Statistical Offices of participating countries.

An MSE estimation strategy will be developed using the parametric bootstrap sampling method. In this context, the logit-transformed area-level model for estimating the QRI will be compared with the corresponding model for the Gini coefficient, as proposed by Runge (2023).

The Gini coefficient will be used as a benchmark due to its status as the most widely used measure of income inequality and its extensive study in the literature. Other quantile-based inequality indicators will be excluded from comparative studies because they lack bounded support, requiring different model assumptions. Additionally, these indicators focus primarily on the distribution tails, making them less informative. Their estimators tend to exhibit a substantial bias in small samples, rendering them unsuitable for the unbiasedness properties assumed in area-level models.

The study will aim to generate inequality maps to reveal distinct patterns between the two indicators, demonstrating how each captures different aspects of inequality. Specifically, it is expected that the QRI will show greater sensitivity in identifying disparities between domains, even when the measured inequality level is low or when there are no extreme values that could distort the comparison of inequality levels when using the Gini coefficient.